Fractal dimensions, scaling, and bifurcation in the Solar System

Vladimir Kuznetsov

Institute of Space Physics Research DVO RAS, Kamchatka, Russia
E-mail: vvk@ikir.kamchatka.ru

Abstract

Fractal dimensions, scaling, and bifurcation in the Solar System bear evidence of self-organization at the moment of its formation. Fractal dimensions (*D*) defined by planetary distance (*R*) vs. planet number (*n*) logarithmic relationships ($D = lgR/lgn$), are the same (1.75) for the terrestrial (inner) planets and increase with ***n*** for the giant outer planets from 1.97 in the asteroid belt to 2.6 in Neptune. The behavior of the *D*(*n*) function shows that the inner and outer planets formed in different conditions. The process of the Solar System formation must have undergone abrupt bifurcation in the region of the asteroid belt whereby the protosolar disc split into several disks and the formation of iron-rocky planet cores proceeded within each disk, by the same mechanism that produced the Sun. The Sun, the planets, and their satellites apparently formed in a single process, by the "hot" mechanism, and the formation of the planets and the satellites was only slightly longer than the Sun formation.

Keywords: Solar System forming; fractal dimensions; scaling; bifurcation.

1. Introduction

The origin of the Sun, the Earth, and other planets and their satellites has been one of the most exciting puzzles since the ancient times. Understanding the formation mechanism is more so important that it controls the physical properties of planets and their evolution. The planetary formation scenario accepted through the past fifty years implies that the Earth formed gradually by collisional accumulation of successively larger solid bodies, planetesimals, in the protoplanetary cloud. It is referred to as a "cold mechanism", because the released heat is radiated into the space. An alternative possibility is that planets, including the Earth, contract rapidly by self-gravity and heat up by the heat stored in their interiors. The Earth, "made" in this way, would heat up to ~ 30,000 K (Magnitsky 1965); the temperature of Jupiter estimated by the same method as used in (Magnitsky 1965) would be of the order of 300,000 K, etc.

Although generally assumed, the cold formation of the Solar System has never been proven valid by astronomical or astrophysical experiments. The cold mechanism was first doubted after the space vehicles Pioneer-X and Pioneer-XI discovered strong heat emission from Jupiter (Kozyrev 1977) and high symmetry of its gravity field typical of a gas sphere. The temperature in the planet's core was inferred from heat flux to reach 165,000 K (Kozyrev 1977), which is about twice as low as the 300,000 K obtained as in(Allard 1995). These discoveries inspired a discussion whether Jupiter is a planet or a star. Some (Boss 2002) believe that Jupiter and the other giant planets may have hot interiors. More doubts about cold planet formation come from the discovery of brown dwarfs, infrared stars intermediate in mass between Jupiter and the Sun (Allard 1995; Nelson 1995; Rebolo et al. 1995). It appears paradoxical that Jupiter, with liquid or even solid hydrogen in its interior, would be a cold planet and a brown dwarf, just ten times the jovial mass, would be a star, though infrared. Eventually, unexpected opacity profiles of young million-year-old circumstellar disks in the Orion nebula obtained recently with the ***Hubble Space Telescope*** (Throop et al. 2001) imply planetary formation models other than the standard for giant planets such as Jupiter.

The new evidence for possible presence of planets in other stellar systems, especially the HST data on circumstellar disks, the "birthplace" of planets, has stimulated growing attention to the problem 'what is a planet?' The discussion of this question in *Science* (McCaughrean et al. 2001) and in Internet (Lissauer 2001) showed that no unique answer was so far possible. This uncertainty may be, among other reasons, due to the conflict of the "cold" formation mechanism with the recent astronomic evidence.

2. Fractal dimensions of the Solar System

Orbital distances of planets in the Solar System follow the Titius-Bode law (Melchior 1947; Nieto 1972):

$$R = 0.4 + 0.3\times 2^{n}, \qquad (1)$$

a scaling relationship where R is distance from the Sun in AU and n is integer ($n = -\infty$ for Mercury, $n = 0$ for Venus, $n = 1$ for the Earth, etc.). The distances from the planets to their satellites in the systems of Jupiter, Saturn, and Uranus obey the same law, which may indicate that the Solar System and the planet-satellite systems formed by the same mechanism.

May the Solar System be referred to as a self-organizing fractal structure once it is postulated to involve scaling? The positive answer would allow a new insight into its origin. Space and time self-organization in the Solar System manifests itself in fractal properties of its structure. Self-organization as a rule occurs as the interplay of two processes, such as diffusion and percolation. Revealing these two processes in the Solar System would characterize it as a fractal structure. Self-organization in the Solar System can be investigated using two parameters – the planet number n and the distance from the Sun R – in terms of Hausdorff's fractal dimension D_H (Mandelbrot 1983):

$$D_H = -\lim \frac{\ln N(r)}{\ln(1/r)}, \qquad (2)$$

where N is the number of events and r is their energy. The Solar System would be said a fractal structure if the planet number and the planetary distances were related as

$$R \approx A + n^{D}, \qquad (3)$$

where D is non-integer and A is constant.

The planetary distances R in AU×10 as a function of planet numbers n in the Solar System are shown in Fig. 1 together with the fractal dimension of this relationship, estimated as

$$D \approx \lg R/ \lg n, \text{ where } R_{(AU)} \approx 0.1(3 + n^{D}), \qquad (4)$$

Note that the planets fall into two distinct groups: all terrestrial planets (Mercury, Venus, Earth, and Mars) have the same $D = 1.75$ and in the others D increases from 1.97 in the asteroid belt to 2.6 in Neptune and Pluto. Lower D values are assumed to correspond to a higher degree of self-organization, and the fractal dimension of a self-organized structure is believed to decrease with its evolution. The same fractal dimension in the four terrestrial planets may indicate that they have lived through the same evolution.

The Solar System is a two-dimensional structure, and the non-integer dimensions we have obtained mean that the system is fractal. The $D < 2$ for the terrestrial planets shows a higher degree of their self-organization than in the outer planets. This fact may mean that the terrestrial planets formed in a process which determined their position relative to the Sun and may have been driven by mass transport from the planets to the Sun. The originally single process of the Solar System formation must have been interrupted by bifurcation (e.g., density redistribution) in the region of the asteroid belt. Thus, the division into inner and outer planets is not fortuitous (Fig. 1).

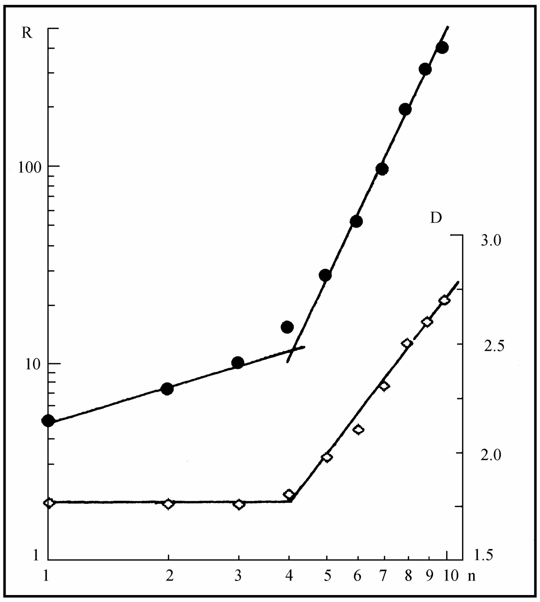

Figure 1. Distances from Sun *R* in AU×10 (bold circles) and fractal dimensions *D* (open diamonds) vs. planet number *n*, in logarithmic scale. 1- Mercury, 2 - Venus, 3 – Earth, 4 - Mars, 5 - asteroid belt, 6 - Jupiter, etc.

3. Density distribution in the Solar System

Bifurcation in the Solar System can be explained in terms of density distribution. It appears reasonable to estimate surface rather than volumetric density in this two-dimensional structure in which the planets orbit the Sun in the ecliptic plane coinciding with the Sun's equatorial plane. Each planet, including the asteroid belt, most likely formed of the disk material confined between its radius (R_n) and the radius of its neighbor planet (n-1). Dividing the planet mass by the area of the protoplanetary disk gives the surface density:

$$\rho(R) = M_n/\pi(R^2_n - R^2_{n-1}). \qquad (5)$$

In Figure 2 surface density of the Sun (S) and the planets is plotted against orbital distances *R* (in logarithmic scale): Mercury (1), Venus (2), Earth (3), etc. The behavior of $\rho(R)$ for Jupiter (6), Saturn (7), Uranus (8), and Neptune (9) appears meaningful and fits the Gaussian distribution. Assume that at $R = R_n$, $n = 6, .. 9$, this relationship reflects density distribution in the protosolar disk at the origin of the Sun and the planetary system before bifurcation (density redistribution) and the extension of the line linking the points 9, 8, 7, and 6 from Neptune to the Sun (S) represents the initial density distribution in the protosolar disk. The points 5', 4', 3', 2', and 1' correspond to $\rho(R)$ on the orbits of the asteroid belt and the terrestrial planets before bifurcation. According to our assumption of the Gaussian density distribution in the disk, the latter would have generated a brown dwarf instead of the asteroid belt if bifurcation had occurred closer to the Sun, say, in the region of Mars (4'). If it had happened a year later and got on the Earth's orbit (3'), another star would have formed instead of Mars, the Solar System would have become a double star as 90 % of our Galaxy, and no such planet good for life as the Earth would have emerged.

Bifurcation can be localized proceeding from a straightforward relationship of gravity potential. Density redistribution is possible at the distance R from the Sun where the Sun gravity potential GM_S/R (M_S is the solar mass) equals or exceeds the proper potential of the forming planet of the mass M_p and radius R_p (GM_S/R ³ GM_p/R_p). For instance, the gravity potential of the Sun GM_S/R at the distance R from the Earth is 6×10^{12} (cm/s)^2, whereas the potential of the Earth is 6×10^{11} (cm/s)^2; for Jupiter $GM_S/R = 10^{12}$ (cm/s)^2 and its potential is $GM_p/R_p = 2\times10^{13}$ (cm/s)^2; for the asteroid belt $GM_S/R = GM_p/R_p \approx 2\times10^{12}$ (cm/s)^2. Then, it is easy to predict the M_p/R_p ratio of the failed planet in place of the asteroid belt. The position of the bifurcation point is thus controlled by the parameters M_S, R, M_p and R_p at which the equation fulfills.

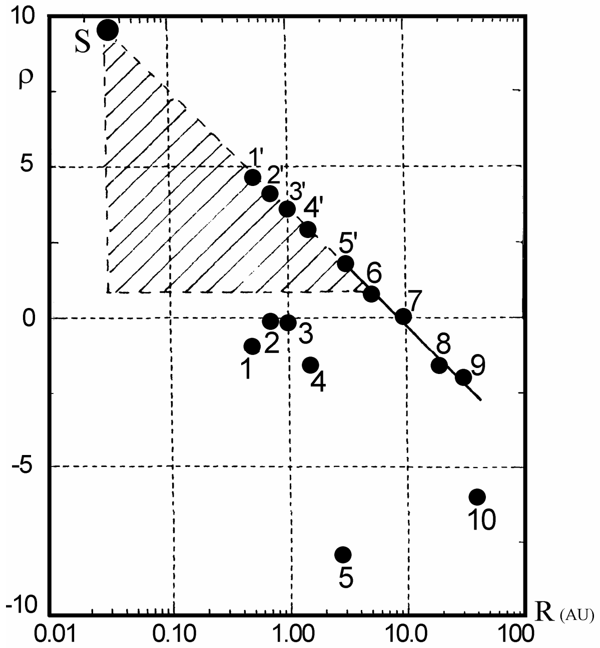

Figure 2. Surface density of Solar System planets (1', 2', 3', etc.) as a function of their orbital distances R, in logarithmic scale. Numerals correspond to planet numbers in Fig. 1. Mass of hatched triangle corresponds to solar mass. Surface density (1', 2', 3', etc.) corresponds to density prior to bifurcation.

4. Disk contraction and Jeans criterion

In his studies of evolution and stability of stars Jeans showed that self-gravitation of stars and their ensuing contraction are impeded by the pressure of interstellar gas. Then density increase is paradoxically associated with the respective increase in both the self-gravity and the impeding inner gas pressure. The self-gravity – gas pressure balance is regulated by the Jeans criterion requiring that density exceeded some critical value ρ_c at which gravity forces are equilibrated with inner pressure (Jeans 1916). The critical density ($\rho_c = m_c\ m\ \ m_P$, where m is the atomic weight, and m_P is the proton mass) of a circumstellar disk of the mass M and the temperature T is expressed via the mass m sub c, which is (Lin et al. 1969):

$$m_c = 10^3\ [T^3/(M/M_S)]\ \text{cm}^{-3}, \qquad (6)$$

a typical large diffuse cloud may have the mass $M = 10^3\ M_S$ and T = 100 K. Then, the concentration of particles in the cloud $m \approx 10$ cm^{-3} is as low as $10^{-2}\ m_c \approx 10^3$ cm^{-3}.
Protostars collapse at the mass

$$M_{Jn} \geq 1/795\ (R_g T/G)^{3/2}\ \rho^{-1/2}, \quad (7)$$

where R_g is the gas constant, G is the gravity constant, and ρ is the density of the disk.
The Jeans critical density is $\rho_c = M_n/\ \lambda^2$, where λ is the Jeans wavelength, $\lambda = R_n - R_{n-1}$. The Jeans critical mass M_{Jn} required for the onset of gravity contraction is given by

$$M_{Jn} = \rho\lambda^2 = M_n\,(R_n - R_{n-1})^2/\pi(R^2_n - R^2_{n-1}); \quad (8)$$

wherefrom

$$M_{Jn}/M_n \text{ » } (R_n - R_{n-1})\,/\,(R_n + R_{n-1}). \quad (9)$$

The condition of self-gravity (9) is fulfilled for Jupiter, Saturn, Uranus, and Neptune, but not for the terrestrial planets with their mass much below the Jeans limit: M_{Jn}/M_n for the Earth, Mars, and Jupiter are respectively 10^4, 10^5, and 0.3. It follows from Fig. 2 that the orbital distances R and the densities ρ of Jupiter, Saturn, Uranus, and Neptune are related as $\rho = A\ R^{-b}$, where $b \approx 3.55$ is the power and A is constant. But this relationship breaks down in the region of the asteroid belt $n = 5$, where the fragmentation of the protosolar disk was interrupted by bifurcation. After the bifurcation Jupiter, Saturn, Uranus, and Neptune continued to form by contraction in the disks that were separated from the protosolar disk and developed the systems of satellites similar to the solar planets. The terrestrial planets had time only to form their iron-rocky cores by the moment of bifurcation, and the remainder mass of the disk (mostly hydrogen) was spent to build the Sun. The hatched triangle in Fig. 2 corresponds to the portion of the protosolar disk used to form the Sun of the mass M_S. This mass can be found as

$$M_S = \int_r^R x\rho dx, \quad (10)$$

where x is the size of the proto-Sun. With the density $\rho(x) = A\ x^{-b}$ (x instead of R), M_S is

$$M_S = A\int_r^R x\,x^{-b}\,dx = A[(x^{-b+2}/(-b+2)]\ \Big|^R_r\,. \quad (11)$$

Substituting R and r gives

$$M_S \approx A\ r^2/(b - 2)\ r^{\,b}, \quad (12)$$

where R is the orbital distance of the asteroid belt, and r is the radius of the proto-Sun at the moment of bifurcation; n was inferred from the slope ($b = 3.55$). A can be found by substituting the jovial mass M_J and the orbital distance R_J:

$$A = R^b{}_J\,M_J\,/\pi R^2{}_J = (1/1.55\pi)\ (R_J{}^{1.55}\ M_J). \quad (13)$$

Substitution of A into the equation for M_S gives

$$M_S/M_J \approx 1/5\ (R/r)^{1.55}, \quad (14)$$

where r is the radius of the forming Sun, which is 2.5×10^6 km, or about three times the modern solar radius. Therefore, bifurcation occurred when the Sun was forming and had a radius three times as great as now.

5. A possible formation mechanism for the Solar System

Star formation in galaxies has been widely believed to be a consequence of some wave mechanism generating density waves. Lin and Shu (Lin et al. 1969; Roberts 1969) developed a density-wave theory in which the wave pattern shows a spiral structure. Their model includes compressional waves – similar to acoustic density waves in gas – that are generated in the center of a spiral galaxy and propagate along spiral arms, and star-forming regions arise in the denser wave loops.

Density waves may have emerged in the same way inside the protosolar disk at the moment of the Sun formation. At least, their generation does not contradict the model of Lin and Shu which has no scale limitations on the applicability of the density-wave mechanism to the formation of the Sun and, likewise, the giant planets.

Thus, the here proposed model of the self-gravity formation of the Solar System, including the systems of Jupiter, Saturn, and Uranus, assumes the action of density waves in the protosolar (protoplanetary) disks, where the wave pattern controlled the position of the planets orbiting the Sun and the satellites orbiting the planets.

The model stems from the following premises:

- density distribution in the protosolar (protoplanetary) disk and the kinetic energy of its steady rotation provided the planets, after their formation, with the respective orbital velocities;
- the disk had a density and a temperature high enough to initiate self-gravitation, and the particles were deposited on the core (the Sun) at a gravity (Alfven) velocity greater than that required by the Jeans criterion (Jeans 1916);
- the composition of the forming planets corresponded to the bulk chemistry of the disk material;
- density distribution in the disk changed abruptly at the moment of the Sun formation, i.e., the process was interrupted by bifurcation;
- self-gravitation of the planets (the satellites) proceeded by contraction and consolidation of the disk resulting from gravity perturbation associated with the Sun (planet) formation;
- density distribution in the parent disk as a function of orbital distance corresponded to that prior to bifurcation.

In addition, it should be noted that (i) the growth rate of the consolidated disk core (dr/dt) increases with its size by the scaling law, and (ii) if the Solar System and the satellite systems of Jupiter, Saturn, and Uranus formed by the same mechanism, the lower mass limit of a planet that develops a system of regular satellites would be ≈15 M_E (M_E is Earth's mass). Uranus, which is 14.5 M_E, is orbited by regular satellites but Neptune (17.2 M_E) is not, possibly, because the disk in the region of its formation had a much lower density than in the region of Uranus (Fig. 2). The same reason may have caused the formation of the Moon in the vicinity of the Earth, and Triton near Neptune.

The formation time of a star, a planet, or a satellite can be estimated under the assumption that its iron-rocky core started to form at some moment in the protoplanetary disk. Modeling simulates the process of coagulation, i.e., the growth of a drop accreted by a gas particle sedimenting on its surface. Let the collision of a gas particle with a drop (an embryo of a star, a planet, or a satellite) take the characteristic time t: $t = (\sigma\, k\, v)^{-1}$, where $\sigma = \pi\, r^2$ is the cross section of the drop, $k = \rho/m$ is the concentration of particles in the gas, m is the mass of the particle, v is its velocity, $r = (V_d)^{1/3}$ is the radius of the drop, $V_d = V_p\, g$ is the volume of the drop, V_p is the volume of the particle, and g is the number of particles in the drop.

The growth time is:

$$t = \int dg/\sigma\, k\, v, \qquad (15)$$

as $\sigma = \pi(V_p\, g)^{2/3}$, $t = g^{1/3} / (V_p^{2/3}\, k\, v)$, or $t \approx r/(V_p\, k\, v)$. Substituting the value of k and taking $m/V_p = \rho_{pl}$, the density of the planet material ($\rho_{pl} = 3\ g/\text{cm}^3$) gives:

$$t = (\rho_{pl}/\rho_o) \times (r/v), \qquad (16)$$

where ρ_o is the density of the protoplanetary disk. From (16) it follows that the formation time is proportional to the size of the body and is inversely proportional to the density of the protoplanetary disk and to the drift velocity of its particles. With the velocity $v = (GM/R)^{1/2}$ and the density ρ_{pl}, it is:

$$t_{\min} = 1/\rho_o \times (\rho_{pl}/4\pi G)^{1/2}. \qquad (17)$$

With the known values of G and ρ_{pl}, it is

$$t_{\min} \approx 10^{-4}/\rho_o. \qquad (18)$$

Then, the growth rate of the drop dr/dt is found as:

$$dr/dt = r\,\rho_o\,(4\pi G/\rho_{pl})^{1/2} \qquad (19)$$

Therefore, as it was assumed, the formation of the Sun and the planets involved (i) contraction and core formation, (ii) rotation of the protosolar disk, and (iii) generation of density waves and formation of secondary protoplanetary disks in their loops. The action of this scaling mechanism was interrupted by bifurcation, which inhibited the formation of planet Faitone in place of the asteroid belt, and the inner planets became much less massive than the outer planets. The bifurcation may have been associated with changes in the contraction regime during the Sun formation (see above).

The frequency of density waves shows a weak (decreasing) dependence on distance from the Sun R, which can be illustrated as follows. Let density waves with the frequency ω, similar to acoustic waves, arise in the contracting protosolar disk, and their velocity c_s be related to the pressure p and the density ρ as $c_s \sim (\gamma p/\rho)^{1/2}$ (γ is the adiabatic constant). Let p be proportional to $1/R$ and ρ be proportional to $1/R^2$, then $c_s \sim R^{1/2}$. Earlier we assumed that $\lambda \sim R$, then $\omega = c_s/\lambda \sim R^{-1/2}$. The dependence $\omega \sim R^{-1/2}$ indicates differential rotation of the disk, which appears natural taking into account its viscosity.

6. Faitone: A failed planet

Planets with masses below the Jeans limit cannot form by self-gravity (self-contraction). However, the onset of contraction appears quite plausible if we take into account that the initial density of the separated protoplanetary disk was much above that obtained by dividing the planet mass by the disk area (e.*g*., for the Earth this density was about four orders of magnitude higher: points 3 and 3' in Fig. 2). It is important that bifurcation started after the onset but before the end of self-contraction, when the iron-rocky cores of the future planets had already formed. Why then Faitone should be a failed planet, has not been the initial density of its parent disk above the Jeans limit? It appears that at least a small planet, even smaller than Mars, could have formed in the region of the asteroid belt. Apparently, self-contraction did occur at its orbit but some other reason may have caused the formation of asteroids instead of a planet.

Here another criterion of planet formation comes into play, namely, the formation time of the protoplanetary disk controlled by the velocity v_R of a particle that drifts along a disk of the radius R (R is the distance from the Sun): $t_R = 2\pi R/v_R$, then $t_R \sim R$. Planet formation obviously requires that $t_\rho > t_R$, where t_ρ was estimated in (18) as $t_\rho \approx 10^{-4}/\rho_o$. For convenience, express t_R via the density $\rho_R \sim 1/t_R$, or $\sim 1/R^2$. The condition $t_\rho > t_R$ may have broken down just in the region of the asteroid belt. It follows from Fig. 2 that the line ρ_R is below the point 5' corresponding to the density of the disk before the onset of bifurcation. This or some other similar mechanism, when the continuous process was broken in the planet-forming region, may have inhibited the formation of Faitone in the Solar System. The change in the contraction regime and the breakup of disk continuity were followed by rapid compression collapse and the formation of the Sun. The density distribution in the disk changed as well, and the smooth initial function (extension of the line 9 – 6 in Fig. 2) gave way to density decrease towards the Sun (points 4 – 1 in Fig. 2). The densities of the Earth, Mars, and the asteroid belt ($\rho_o = 10^{-8}$, 2×10^{-8}, 2×10^{-8} g/cm^3, respectively) were used to estimate t_ρ and t_R (in years), which are, respectively, 10^4, 5×10^3, 5×10^3 and 3×10^3, 4.8×10^3, and 8.4×10^3. Hence, the condition $t_\rho > t_R$ fails in the asteroid belt. The breakdown of this condition may have inhibited the formation of Faitone. In the Mars disk it fulfilled only partially ($t_\rho \approx t_R$), and this might be the reason why Mars is so small (Fig. 3). At the same time, the start of self-contraction prior to bifurcation allowed the formation of the small constituents of the asteroid belt but not a normal iron-rocky core of a planet.

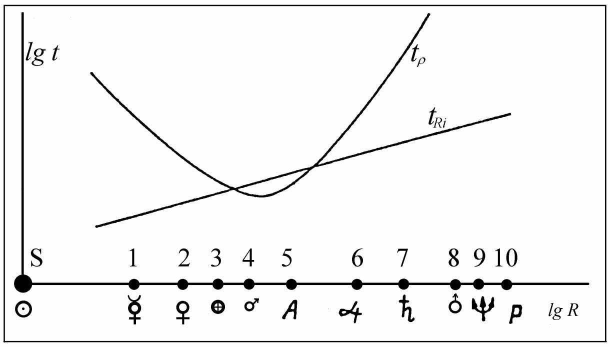

Figure 3. Formation times of a planet (t_ρ) and a protoplanetary disk ($t_{R\ i}$) as a function of orbital distances (R). 1 – Mercury, 2 – Venus, etc.

7. Unified sequence “Star – Planet – Satellite”

Thus, our model stems from the postulate that the Sun, the planets, and their satellites formed by the same scenario in the single process, with the only difference in the amount of primordial material involved in this process.

It is generally accepted that stars exist by nuclear burning of hydrogen, and the mass of a star able to sustain the thermonuclear fusion must be at least 1/25 solar mass. What are then the less massive objects? Jupiter has 0.001 solar mass, and brown dwarfs, infrared stars, are 10 times Jupiter's mass, i.e., they bridge the gap between stars and planets. The approximate logarithmic mass dependence of temperature for a star, a planet, and a brown dwarf (Fig. 4) shows that satellites, planets, and brown dwarfs may continue the main star sequence, in which the difference is based on the amount of material involved in formation and, hence, the temperature.

Temperature (T in Fig. 4) means the temperature in the core of a star, a planet or a satellite, where it remains almost the same (only slightly lower) as at the moment of formation. This temperature can be estimated as T » $3/5\ GM/Rc_p$, where c_p is the specific heat, G is the gravity constant, and R is the radius of the body. If, in the first approximation, specific heat and density are assumed constant for all bodies, temperature and mass are related as $T \sim M^{2/3}$, which is valid for the stars, brown dwarfs, planets and their major satellites. Note that the value c_p increases with temperature, and the temperatures found in this way are slightly overestimated.

A similar dependence can be obtained for the stars of the main sequence by superposition of the temperature and mass dependences of luminosity of a star from (Chandrasekhar 1939; Kuznetsov 1998; Kuznetsov 2000) (Fig. 4), assuming the surface temperature uniquely dependent on the core temperature (though this is not quite obvious).

Note that all bodies in the Solar System, sized from the Sun to Europe, Jupiter's satellite, are spherical. The gravity potential GM/R of Europe is about 2 kJ/g, which with a reasonable specific heat of 0.3 cal/g defines its temperature as 1.5×10^3 K. All other bodies (including asteroids and satellites) with the masses below Europe's mass are of irregular shapes and, probably, have not undergone the stage of total fusion. The bodies of the Solar System can be divided into three groups on the basis of their gravity potential ratios relative to the heat associated with phase transition (evaporation or condensation and melting or crystallization). If we assume that the heat released in evaporation of SiO_2, the basic material of the terrestrial planets, is U_e = 15 kJ/*g* and that of melting is U_m = 1 kJ/*g*, $GM/R < U_m$ for small satellites and asteroids, $U_m < GM/R < U_e$ for large satellites and small planets (less than Venus), and $GM/R > U_e$ for the planets more massive than Venus (Fig. 4).

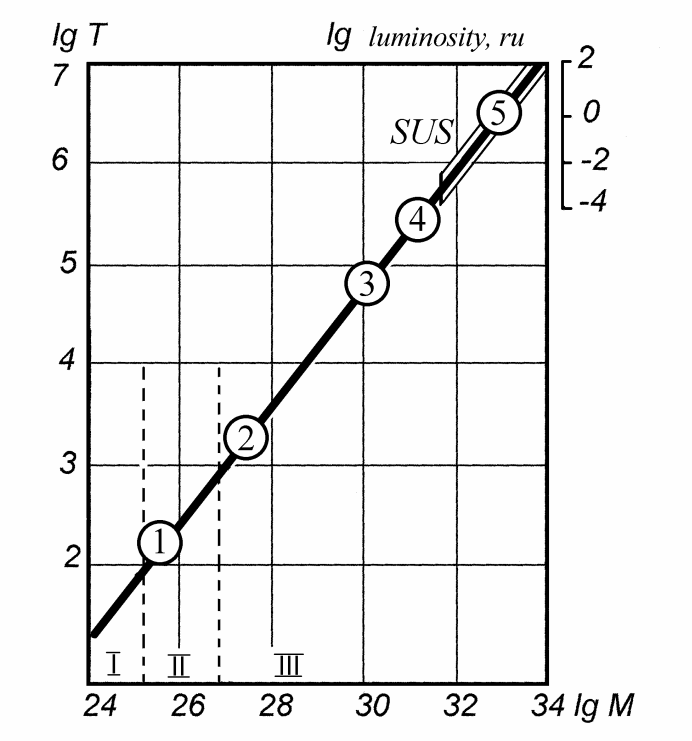

Figure 4. Logarithmic mass dependence of temperature in the cores of stars and planets (Sun (5), Brown Dwarfs (4), Jupiter (3), Earth (2), and Moon (1)) and relative mass dependence of luminosity of stars in main sequence (top right corner). I, II, and III are three groups of bodies in the Solar System.

8. Discussion

The Solar System at the moment of its formation was an open self-organizing structure in which the Sun, the planets, and their satellites originated in a single process. This approach defines the characteristic time of the process (not very different from that for stars, no longer than 1 million yr) and the hot origin mechanism for the star, the planets, and the satellites.

Open self-organizing systems are maintained by the interplay of two alternative processes, such as diffusion and percolation. In our model, consolidation in loops of density waves propagating peripheryward in the protosolar disk and self-gravity contraction interact with mass transport toward the star (the Sun) forming in the disk center. The more efficient the interaction of the two mechanisms, the more efficient the self-organization and the lower the fractal dimension. The lower value of D for the terrestrial planets is evidence of "more favorable" formation conditions than for the gas giant planets. The gradual growth of D from 1.75 to 2.6 also indicates that all planets of the Solar System formed in a single process. The decrease in the density of mass flow away from the Sun is obvious, and may have determined the increase in D with R. The fact that the mass "withdrawn" from terrestrial planets is exactly the solar mass likewise supports the single formation scenario.

Note that the interpretation of the self-gravity (self-contraction) concept appears ambiguous. Lin and Shu (Lin et al. 1969; Roberts 1969) invoke self-gravity as the only way of star formation in the loops of density waves in galaxies. The initiation of self-gravitation requires the fulfillment of the Jeans criterion. However, it remains unclear what happens if self-gravitation has started but the amount of material decreases, for instance, being transported towards the forming star. Our model may be quite plausible if we assume that self-gravitation has initiated with the formation of iron-rocky cores (it is commonly accepted that Jupiter and other giant planets have the cores of this kind) and this process cannot stop anyway.

Hypothesizing that the Earth and the Solar System are made of the same material we left aside the problem how the rocky silicate Earth can have formed from a gas cloud. This problem was discussed earlier in terms of the "hot Earth" model (Kuznetsov 2000). According to that model (Kuznetsov 2000), the inner core of the Earth contains the same matter that composes the Sun and the planets, large or small, and this material occurs in a condensed (fluid or solid) state in the other Earth's rinds, in the same proportion as in the Sun, without light gases (hydrogen, oxygen, nitrogen, etc.).

Our calculations do not include the rotation of the protosolar disk with the forming Sun and the planets, which have preserved the rotation in the ecliptic plane and set into rotation round their axes. The reason is that it remains unclear why all gravitating bodies, from satellites to the Universe, rotate, though rotation is obviously a fundamental property of nature. The regard for tidal effects will complicate the model but will not add much to the key idea of the Solar System formation, as slow rotation does not contribute notably into self-gravitation (Votyakov et al. 2002).

There remains another problem relevant to the discussion of planet formation: Can the mass flow from the bifurcation point to the Sun collide with an orbiting planet and push it off the orbit? The improbability of this event can be illustrated by a straightforward calculation for Mercury, which is, first, the smallest planet especially prone to change its orbit and, second, faces the heaviest flow. The impossibility for Mercury to run off its orbit under the impact of the mass flow will be proven positive once the momentum ratio is proven to be far above unity, or $Mv/mV_m >> 1$ (orbital momentum is much above that of eccentric impact). M is the mass of Mercury ($M \approx \times 10^{26}$ g), v is the orbital velocity ($v \approx 50$ km/s), V_m is the gravity velocity on the Mercury orbit ($V_m \approx 5$ km/s), and m is the flow mass, the only variable to be estimated. The total mass of the flow corresponds to the area of the triangle from the point 6 to the line 1-1' in Fig. 2, which is about 10^{-5} M_S, taking into account that the whole triangle corresponds to M_S (see above). The portion of the 10^{-5} M_S flow that may collide with Mercury can be estimated using the thickness of the protosolar disk H, assumed to equal the diameter of the proto-Sun at the moment of bifurcation ($H = 5 \times 10^6$ km). This portion, found as the ratio of the Mercury surface area (pr^2_M) and the flow section area ($2pR_1 ´H$), where R_1 is the orbital distance of Mercury (60×10^6 km) and r_M is its radius ($r_M \approx 2.4 \times 10^3$ km), is $\approx 10^{-8}$. Then, the momentum ratio is $Mv/mV_m \approx 10^7$ or much above unity, *quod erat demonstrandum*.

Although the flow causes a very little effect on the orbital motion of the terrestrial planets, it, in interaction with the wave density mechanism, corrects the orbits so that the fractal dimension of the system decreases and its self-organization increases. This interaction appears to be based on changes in the parameters of the latter rather than on elastic collision of particles with planets.

9. **Conclusion**

The proposed model is consistent with three postulates mentioned in the beginning:

1. Like other giant planets, Jupiter is a hot planet. (Its temperature may be maintained, for instance, by recombination of hydrogen ionized during the planet formation);

2. Brown dwarfs are natural objects that bridge the gap between stars and giant gas planets. (Judging by the predicted temperatures of their interiors, they may involve synthesis of hydrogen from protons and neutrons);

3. Planets may develop their iron-rocky cores faster than in 1 myr and much faster than it has been commonly believed (e.g., in about 10,000 instead of 100,000,000 years for the Earth).

The proposed formation model of the Earth and the Solar System as a whole cannot be free from some ambiguities or contradictions (at least because, according to Goedel's theorem, exhaustiveness cannot coexist with consistency within one logical system). Nevertheless, its key postulate that all elements of the Solar System (the star, the planets and their satellites) formed simultaneously and in a single process calls for revision of the previous approach to the origin of stars, brown dwarfs, large and small planets, and satellites.

References

Allard F., 1995, Nature 378, 441

Boss A.P., EPSL 2002, 202, 513

Chandrasekhar S., 1939, An introduction to the study of stellar structure. Oxford University Press. London.

Jeans J.H., 1916, Roy. Soc. London Proc. A, 93, 413

Kozyrev N.A., 1977, Astronomical Journal 54, 372 (in Russian)

Kuznetsov V.V., 1988, Geology and Geophysics 39, 993

Kuznetsov V.V., 2000, The hot Earth model www.uiggm.nsc.ru/~kuz/site.htm

Lin C.C., Yuan C., Shu F. H., 1969, Astrophys. J. 155, 721

Lissauer J., 2001, SCIENCE Online www.sciencemag.org/cgi/content/full

Magnitsky V.A., 1965, The Earth's inner organization and its physics. Moscow. Nauka. (in Russian)

B.B. Mandelbrot, The Fractal Geometry in Nature. Freeman. New York. 1983.

Mc Caughrean M., Reid N., Tinney C., et al., 2001, Science 291, 1487

Melchior P., 1947, Eléments canoniques des orbites et representations des distances planétaires Acad. Roy. Sci. Lett. Beaux-Arts Belgique, Bull. 33, 212

Nelson L.A., 1995, Nature 377, 102

Nieto M.M., 1972, The Titius-Bode law of planetary distances: its history and theory. Pergamon Press. Oxford

Rebolo R., Zapareto Osorio M. R., Martin L., Nature 377 129

Roberts W.W., 1969, Astrophys. J. 158, 123

Throop H.B., Bally J., Esposito L.W., Mc Caughrean M.J., 2001, Science 292, 1686

Votyakov E.V., Hidmi H.I., De Martino A., Gross D.H.E., 2002, Phys. Rev. Lett. 89, 031101.