Elastic Properties of Natural Dusty Plasmas

V. Kuznetsov

Institute of Geophysics, SB RAS, Novosibirsk,
Institute of space research DVB RAS, Kamchatka, RUSSIA

Abstract

The model of natural plasma crystal has been developed. Mechanical stability of a structure formed by the dust grains in plasmas is explained by exotic force, the Casimir force, holding the negatively charged grains together. Plasma crystal was considered as the Wigner crystal and its elastic properties were calculated.

Keywords: Elastic Properties; Natural Dusty Plasmas;

1. Introduction

Recently, unusual properties of dust grains introduced into a plasma, were considered as a manifestation of a new state of matter named "dusty plasmas" [1]. Dusty plasma is produced after controlled or uncontrolled injection of dust grains into gas excited by electric discharge. The grains treated have a negative charge (Z) equivalent to several hundred electrons and form 3D ordered equidistant structure (plasma crystal) similar to the arrangement of atoms in hexagonal solids. The dust grains range from tenths to tens of microns in size. The distance between the grains is as large as several hundred microns [2].

A dusty plasma cannot be considered as exotic state of matter. This is thought to exist inside a vacuum chamber of an apparatus for thermonuclear synthesis. High-temperature plasma interacts with the walls of the chamber producing a dusty plasma which in turn affects the technical performance of the apparatus. Dusty plasmas in a form of plasma crystals are thought to exist in space forming, for example, (dust nebulae) and planetary rings [4]. The dust grains were found in the vicinity of the Sun manifesting themselves as stable [5]. Dust grains and dusty plasmas affect technological procedures at etching in microelectronics, etc. [4]. Properties of dusty plasmas are the subject of plentiful reports [6-13], conferences, monographs (see references in [11-13]), and scrupulous reviews [4, 14]. None of the reports considers the problem of to what extent the plasma crystal is really "hard" and similar to a "crystal". It is known, that some authors suppose about dusty plasma crystal is Yukawa lattice [15-19] or Wigner crystal [20]. Recently, in observation of shear wave mach cones in 2D dusty plasma crystal was measured of compressional and shear waves speeds [19, 20]. Consequently, the question can be formulated as follows: Are the properties of a plasma crystal similar to those of elastic solids? The answer to the question provides us with the key to the explanation of properties of a matter in star nuclei, brown dwarfs, and planets.

2. Model

Neither theory nor even consistent model of a plasma crystal was created so far. The main problem is the nature of the attraction among negatively charged grains in dusty plasmas. The force must be not less than the Coulomb force that repulses the grains: $f_e = (Ze)^2/R^2$, where e is the elementary charge, R is the distance between the grains. The grains are surrounded by a plasma that shields the Coulomb interaction between grains decreasing the force by a factor of k: $k = exp(-R/l)$, where $l = (kT/4pne^2)^{1/2}$ is the Debye radius of shielding, n is the plasma density. To balance the Coulomb repulsion, a "shadowing" force [6, 7] was supposed to exist. It shields the grains due to the flow of a plasma like it was predicted 200 years ago when LeSage tried to create the theory of gravity using the flow of ether shielding the bulk bodies under interaction. Other idea to find the way of Coulomb forces screening lies in the fact of electric charge screening by plasma [7] and in Yukawa potential surving instead the Coulomb one [15-19]. Both conceptions of among the dust plasma grains attraction occurring through the in screening of plasma flow seem true, except that they have no sufficient theoretical and experimental arguments. Unfortunately, there is no experimental argument in favor of the hypothesis so far.

There has long being published the information about experimental evidence of interaction between bulk solids, other than gravitational, Coulomb, magnetic, etc. This is the phenomenon predicted by Casimir in 1948. It was tested many times in different scientific laboratories [21-24]. The interaction was named the Casimir force. The phenomenon lies in the fact that two conductive metal plates in a vacuum attract to one another exerting a pressure of $f_C = - (p^2/240) (\hbar cS/R^4)$, where R is the distance between the plates, S is the plates square, $\hbar$ is the Planck constant, and c is the speed of light. The force is unusual as it does not depend on mass, charge or other parameters of bodies under interaction.

Before the phenomenon received its name, Lifshitz was the first to develop the theory of molecular attraction between solids as a function of temperature [21]. Afterward, the Casimir force was explained as a result of a change in a spectrum of zero-point vibrational energy of electromagnetic vacuum when the tangential contribution to the electromagnetic field on the plates reduces to zero. Only those properties of the Casimir force were investigated to date that are connected with the zero-point vibrations of electromagnetic field and, in fact, these are equivalent to the delayed Van der Waals forces [22, 23].

At the very beginning of the experiments, it was found that, according to Casimir, metal plates in vacuum attract to one another. Later, the experimentalists investigated the interaction between various solids. The phenomenon was found to exist not only in high vacuum, not only at extremely low temperatures, not only between metals but also between dielectric solids of various shapes [24]. In [25] shown that Casimir forces between dielectric materials with nontrivial magnetic susceptibility perhaps no attraction, but repulsive. Not all possible experimental conditions and experimental procedures for the measurements of the Casimir force were tested. The investigations are running still in different scientific laboratories [26, 27].

3. Governing equations

In this work, the elastic properties of a plasma crystal are derived from the Casimir force f_c, as an attractive force between charged grains, opposed to the Coulomb force repulsing the grains. The potential of interaction among grains in a dusty plasma is used for the calculation of compressibility of a dust structure and, finally, for the evaluation of elastic properties of plasma crystals. To perform the evaluations, we use the approach developed for the calculation of the sound speed in ionic crystals [28]. The energy of a "crystalline lattice" in a plasma crystal is $U = Nj$, where N is the number of pairs of grains, j is the potential. N is used here instead of $2N$ to avoid duplicate calculation of the contribution to the total energy of a crystal U, for every pair consists of two grains.

The potential of a crystalline lattice can be written as $j = S j_{ij}$, where j_{ij} is the sum of the Coulomb and Casimir contributions: $j_{ij} = f_C R_{ij} - e^2/R_{ij}$. Substituting the complete forms of the contributions, we have

$$j = p^2 \hbar cS/240R^3 – [(Ze)^2/R] \exp(-R/l). \tag{1}$$

The bulk modulus of a plasma crystal ($1/b$) is the second derivative of the energy with respect to a volume:

$$1/b = Vd^2U/dV^2 = V[dU/dR \times d^2R/dV^2 + d^2 U/dR^2 \times (dR/dV)^2]. \tag{2}$$

As most experimental works show the hexagonal order among grains in dusty plasmas, the volume of a unit cell should be written as V » $3N \times R^3$. Under equilibrium, the condition for the energy of a plasma crystal is $dU/dR = 0$. Considering that

$$(dR/dV)^2 = 1/81\ N^2 R^4, \tag{3}$$

and inserting the equation for j, we receive

$$U = N \{p^2 \hbar cS/240R^3 – [(Ze)^2/R] \exp(-R/l)\}, \tag{4}$$

from here, replace: $C_1 = p^2 \hbar cS/240$, $C_2 = (Ze)^2$, $C_3 = 1/l$, have;

$$U = N [C_1 1/R^3 – C_2/R \exp (-RC_3)], \tag{5}$$

and differentiating (5):

$$dU/dR = N[- 3C_1 \times 1/R^4 + C_2 (1/R^2 + C_3/R) \exp (- RC_3)], \tag{6}$$

$$d^2U/dR^2 = N [12C_1/R^5 – C_2 \exp(-RC_3) (2C_3/R^2 + C_3^2/R + 2/R^3)]. \tag{7}$$

Take into account equilibrium condition ($dU/dR = 0$), to have $C_2 \exp (-RC_3)$ from (6):

$$C_2 \exp (-RC_3) = (3 C_1/R^4) / (1/R^2 + C_3/R). \tag{8}$$

Put (8) under (7), to have the second derivative of the energy with respect to the volume:

$$d^2U/dR^2 = N [12C_1/R^5 - (3 C_1/R^4) (2C_3/R^2 + C_3^2/R + 2/R^3) / (1/R^2 + C_3/R)],$$

Taking into account, that $RC_3 \approx 1$ ($R \approx l$), simplify the formula to have:

$$d^2U/dR^2 » N [12C_1/R^5 - (3 C_1/R^4) \times 5/2R], \tag{9}$$

if:

$$d^2U/dR^2 » N [12C_1/R^5 – 15/2 (C_1/R^5)],$$

then bulk modulus of a plasma crystal ($1/b$) is equal:

$$1/b » 10^{-3} p^2 \hbar cS/R^6. \tag{10}$$

Account the electromagnetic nature of the processes under research, change $hc = e^2/a$ at (10), here a is a fine structure ($a = 1/137$) to have the net formula:

$$1/b \;»\; p^2 (Ze)^2/10^3 aR^4. \qquad (11)$$

The evaluation of the interaction area *S* of the dusty grains at (10) requires $S \approx l^2 \approx R^2$ rather than the real size of a grain in the context of electric charge screening and of the Yukawa potential, surveying instead the Coulomb one, under the electromagnetic interaction between grains

Let us compare this formula with the similar formula for the bulk modulus (*1/b*) of metals [28]:

$$1/b \;»\; A\,(Ze)^2/18R^4,$$

here *A* is the Madelung constant.

The "Madelung constant" for our model is: $A = p^2\, 18\,/10^3 a \approx 25$. (For comparison with NaCl: $1/b = 3.3 \cdot 10^{11}$ dyn/cm^2, Madelung constant $A = 1.75$, $R = 2.8\ 10^{-8}$ cm).

4. Elastic properties

The experimental data on [20, 21] are used for the evaluation of the elastic properties of the dusty plasmas. The experiments are concerned to R » $l \approx 0.8$ mm; $Q = Z e \approx 10000\ e$; the grain mass is $m = 5\ 10^{-10}$ *g*; its diameter is 8.7 *mm*. In my view, the chief result of this experiment was that the authors have measured the pulses of compressional and shear waves were launched in the lattice to measure the sound speeds. The sound speeds of the two modes were measured to be $C_L = 24.2 \pm 1.7$ mm/s and $C_T = 5.4 \pm 0.5$ mm/s, for compressional and shear waves, respectively.

The last result permits to estimate the elastic properties of the dusty plasma like the bulk modulus $K = 1/b$ and the Poisson's ratio *s*. Let's use formulas:

$$V_P = [(K + 4/3m)/r]^{1/2};\ V_S = (m/r)^{1/2}, \qquad (12)$$

$$s = (V_p^2 - 2\, V_S^2)/2\,(V_p^2 - V_S^2) = (x^2 - 2)/2(x^2 - 1). \qquad (13)$$

Here the commonly accepted in physics of the solid state designations are used:

$$V_P = C_L;\ V_S = C_T;\ x = V_P/V_S = C_L/C_T.$$

As is shown in [20], $x = 4.48$ thus $s = 0.47$.

As an example, the values of the Poisson's ratio *s* are: 0.28 for iron, 0.5 for liquid, 0.48 for rubber, 0.45 for Earth's inner core.

The estimation of $1/b$ requires the velocity V_P squared and the density of the dusty plasma ρ alike. With the mass of the dusty grains and distance between them, we have r ≈ $5 \cdot 10^{-4}$ g/cm^3. Thus

$$K = 1/b \text{ » } V^2_P \cdot r \approx 5 \cdot 10^{-6} \text{ dyn/cm}^2.$$

The shear modulus of crystal plasma is m » $V^2_S \cdot r \approx 0.25 \cdot 10^{-6}$ dyn/cm^2.

For comparison, let's estimate $1/b$ by (11), supposing: $Z \approx 10000$, $e \approx 4.8\ 10^{-10}$CGSE, $(Ze)^2 \approx 2\ 10^{-11}$, $R = 0.08$ cm, to have:

$$K = 1/b \approx 10^{-6} \text{ dyn/cm}^2.$$

5. Discussion

As we see, the theoretical and experimental values are similar. This conclusion is valid for the assumption of S » R^2. Otherwise, the theoretical estimation would give less value, than the experimental one. Except, the experiments were concerned with 2D plasma, that we didn't take into account at our. The agreement between theoretical and experimental estimations of $1/b$ wouldn't permit the definite conclusion about the solid state of crystal plasma. These are the measurement of shear waves V_S velocities for plasma and the estimation of the value of Poisson's ratio s to the valid supposition of the solid state of crystal plasma. It is known, that solids differ from liquids and gases in their ability to propagate the transverse waves inside them. For the theoretical consideration, it is necessary to make sure that plasma crystals are equivalent to the Wigner crystal [29]. It is believed that the Wigner crystal is capable of propagating the transverse waves and has elastic deformations. Plasma crystal is formed in a dusty plasma if the condition $n_e\, a^3_B << 1$ is valid. Here, $a_B = (2p\ \hbar)^2/m_e e^2$ is the Bohr radius, m_e is the rest mass of electron, e is the elementary charge, n_e is the concentration ($n_e << 10^{25}$ cm^{-3}). Electrons are localized near the equilibrium points, namely the sites of the Wigner cell. The positive charge of the Wigner crystal is distributed uniformly throughout its volume. At first glance, the description of a plasma crystal fits to the features of the Wigner crystal, alike the dense astrophysical plasma [30]. The latter is usually considered as solid crystalline matter, e.g. capable of forming the Wigner crystal, if its criterion of nonideality fits to the condition G

> 178. The criterion *G* equals to the ratio of electrostatic energy of interaction to the thermal energy of a plasma:

$$G = q^2/kT\,R,$$

where $q = Ze$ and R is the distance between particles. The Wigner crystal "melts" if $1 << G < 170$, and a plasma obeyes the laws of the ideal gas if $G \to 0$. In considering a plasma crystal as an ionic one, we can calculate elastic moduli. These can be derived from the experimental data on the speed of acoustic transverse and longitudinal waves propagating throughout the plasma crystal. It is interesting to compare these two sets of elastic moduli and connect them with the criterion of nonideality *G*. Such a comparison is quite reasonable, for the melting of a crystal occurs, on the one hand, under conditions $m = 0$; $s = S$, and, on the other hand, under condition $G < 170$.

In the last works about plasma crystal some authors think that in plasmas Yukawa crystal was origin [15-20]. In the theory used Yukawa inter particle potential

$$j \sim 1/R \exp(-R/l_B), \qquad (15)$$

here l_B is (as in our work) the Debye radius of shielding. However, in the Yukawa potential λ_B is Compton’s wavelength of the intermediate (virtual) boson. As shown [31], inside the mm-region complying closely with our model the boson mass $m_B \leq 10^{-4}\ eV/c^2$ if l_B ~1 mm. If the Yukawa potential agrees with our model, the exchanging bosons between the grains is specified to be, the mechanism of interaction between the Casimir type particles. The Yukawa force similar to the Casimir force, is capable to afford attraction between particles, pushed apart under the effect of Coulomb forces.

6. Conclusion

In conclusion it may be said that it is the Casimir effect to be the basis for generation process of the crystal plasma. The experimental detection of the bulk compression and shear stress in a form of the waves propagating through a dusty plasma as well as the measurements of their speed and the Poisson's ratio are of great importance. Such an experiment is very interesting, for criterion Γ ranges significantly reaching very large values [1] after estimations that are not correct sometimes [4]. The experiments on high-density crystal plasmas at high pressure allows one to develop new approaches to the investigation of plasma physics and physics of condensed matter. Such experiments can give new impetus for the investigation as physical properties of probable components of the Earth's.

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